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THE EFFECT OF THE YIELD STRESS ON THE LAMINAR/TURBULENT TRANSITION

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The laminar/turbulent transition is important for pipe system design as the behaviour of the fluid changes fundamentally at this point. For non-Newtonian fluids there are many different approaches but no guidelines as to which approach is more accurate. The objective of this paper is to show how the yield stress of the fluid affects the different approaches and to show which are more accurate. The yield pseudoplastic theory and the Newtonian approximation, Metzner Reed, Torrance, Bingham plastic, Ryan and Johnson, intersection method and Slatter approaches are reviewed. Analysis is examined from the perspective of the relationship between the critical velocity and the pipe diameter. Behaviour is investigated and horizontal asymptotes at large diameter are identified. The performance of the various approaches are described in general terms. Only the Slatter model can be expected to perform well over the full range of diameters. The importance of the yield stress is emphasised and the controversial aspects of the rheological characterisation procedure are discussed. Future research is outlined.

KEY WORDS: yield stress, transition, non-Newtonian, Reynolds number

NOTATION

Symbol	Description	Unit
A	cross sectional area	m^2
C	constant	
D	internal pipe diameter	m
f	Fanning friction factor	
K	fluid consistency index	$Pa.s^n$
n	flow behaviour index	
Q	volumetric flow rate	m^3/s
r	radius at a point in the pipe	m
R	radius of the pipe	m
Re	Reynolds number	
u	point velocity	m/s
V	average velocity	m/s
Z	stability function	
μ	dynamic viscosity	Pa.s
μ'	apparent dynamic viscosity	Pa.s
ρ	density	kg/m^3
τ	shear stress	Pa
τ_y	yield stress	Pa

Φ	function of

<u>Subscripts</u>

0	at the pipe wall
ann	of the annulus
c	critical
plug	of the plug
shear	over the sheared zone

1 INTRODUCTION

The laminar/turbulent transition is an extremely important piece of information for the pipe system designer because at this point the behaviour of the fluid changes fundamentally. For Newtonian fluids, such as water and oil, the location of the transition is well established and the calculation is trivial. For systems conveying non-Newtonian fluids, however, nothing could be further from the truth. Not only does the literature contain many different approaches, but there are no guidelines as to which approaches are more accurate. The objective of this paper is to show how the yield stress of the fluid affects the different approaches and, from this perspective, to show which are more accurate.

2 THEORY AND LITERATURE REVIEW

2.1 Laminar Flow

Non-Newtonian slurries are often best modelled as yield pseudoplastics (Govier & Aziz, 1972 and Hanks, 1979) and the laminar flow of all the mineral slurries tested by the author have been successfully characterised using the yield pseudoplastic rheological model. The constitutive rheological equation is

$$\tau = \tau_y + K \left[-\frac{du}{dr} \right]^n , \qquad (1)$$

where τ_y is the yield stress
K is the fluid consistency index
n is the flow behaviour index.

The yield stress provides the ordinate offset, and the fluid consistency index and the flow behaviour index together control the rheogram curvature. Equation (1) can be integrated twice to yield the point velocity

$$u = \frac{D}{2\, K^{1/n}\, \tau_0} \frac{n}{n+1} \left[(\tau_0 - \tau_y)^{\frac{n+1}{n}} - (\tau - \tau_y)^{\frac{n+1}{n}} \right] , \qquad (2)$$

and the mean velocity

$$V = \frac{D\ n}{2\ K^{1/n}\ \tau_0^3}\left(\tau_0 - \tau_y\right)^{\frac{1+n}{n}}\left[\frac{(\tau_0-\tau_y)^2}{1+3n}+\frac{2\tau_y(\tau_0-\tau_y)}{1+2n}+\frac{\tau_y^2}{1+n}\right] \tag{3}$$

2.2 Rheological Characterisation

The rheology of the slurries used for this investigation was obtained from laminar tube flow data. The rheological constants (τ_y, K and n) are determined from the data in the laminar region and Equation (3) (Lazarus & Slatter, 1986, 1988 and Slatter, 1994). It is important to note that the above procedure optimises the values of all three rheological constants (τ_y, K and n) for a best fit over the full laminar range available. The value of τ_y is therefore not necessarily the true value at which solid/fluid behaviour changes. However, this value will be the best practical approximation to the yield stress, and is consistent with a pragmatic engineering approach.

2.3 The Laminar/Turbulent Transition

The literature contains many different approaches to determining the critical velocity at which the flow regime changes from laminar to turbulent. Some of these are presented below.

In order to make use of standard Newtonian theory, a value for the viscosity of the fluid is required. Usually the term viscosity is meaningless once a non-Newtonian approach has been adopted. However, an apparent or secant viscosity (Holland, 1973 and Wilson, 1986) can be defined as

$$\mu' = \frac{\tau_0}{\left[-\frac{du}{dr}\right]_0} \ . \tag{4}$$

The Reynolds number may now be calculated using

$$Re_{Newt} = \frac{\rho\ V\ D}{\mu'} \ . \tag{5}$$

Note that μ' is not a constant for a given fluid and pipe diameter, but must be evaluated at a given value for τ_0. The transition criterion is then $Re_{Newt} = 2100$.

Metzner & Reed (1955) developed a generalised Reynolds number for the correlation of non-Newtonian pipe flow data. They defined a non-Newtonian Reynolds number Re_{MR} as

$$Re_{MR} = \frac{16}{f} \quad \text{where} \quad f = \frac{2\ \tau_0}{\rho\ V^2} \tag{6}$$

and τ_0 is evaluated using Equation (3). The transition criterion is then $Re_{MR} = 2100$.

Torrance (1963) based his work on the pseudoplastic model work of Clapp (1961) and investigated the turbulent flow of yield pseudoplastic fluids. He used the following formulation for a Reynolds number, also known as the Clapp Reynolds number (Govier & Aziz, 1972) :

$$Re_{nn} = \frac{8 \rho V^2}{K \left(\frac{8V}{D} \right)^n} . \tag{7}$$

For the same K and n values this Reynolds number will attain the same value for a Pseudoplastic and a Yield Pseudoplastic fluid - the yield stress is totally ignored. It should be noted that there is no direct claim in the literature that this Reynolds number should obtain the value 2100 at the transition point. However, it is included in this work to show the effect of neglecting τ_y and also because of its close association with the turbulent flow of yield pseudoplastic fluids.

A Reynolds number which does take the yield stress into account has been formulated for the Bingham plastic rheological model, for which the flow behaviour index, n, is unity (Govier & Aziz, 1972; Thomas, 1979 and Wilson *et al*, 1992). Assuming that the Reynolds number will be equivalent to 16/f, neglecting the fourth-power term and assuming that the transition from laminar to turbulent flow will occur when $Re_{BP} = 2100$ and then solving for the critical velocity at large diameter,

$$V_c \approx 19 \sqrt{\frac{\tau_y}{\rho}} . \tag{8}$$

An important implication of Equation (8) is that the yield stress can cause the critical velocity to become independent of the pipe diameter at larger diameters. This is in sharp contrast to the Newtonian condition where the product V_cD is a constant. This approach has not yet been extended to the yield pseudoplastic rheological model.

Ryan & Johnson (1959) and Hanks (1981) have derived stability functions for laminar flow velocity vector fields. For axially symmetrical pipe flow the two functions differ by a factor of 2 and only the Ryan and Johnson function will be considered here. The Ryan and Johnson stability function is :

$$Z = \frac{R\, u\, \rho}{\tau_0} \left[-\frac{du}{dr} \right] . \tag{9}$$

For fixed values of R, ρ and τ_0 the Ryan & Johnson function can be regarded as

$$Z = \text{constant} \, . \, u \left[-\frac{du}{dr} \right] , \tag{10}$$

and takes the shape of the product of u and (-du/dr). The maximum value of this function

Z_{max} across a given laminar velocity vector field is taken as the stability criterion. For Newtonian flow, Z_{max} = 808 for Re = 2100 and **it is assumed that all fluids will obtain this value of Z_{max} = 808 at the transition limit.** Although only the Ryan and Johnson criterion is considered in this paper, it must be noted that the analyses and general findings and conclusions applicable to the Ryan and Johnson criterion apply equally to the Hanks approach. Clearly, the stability criterion approach is the most scientifically sophisticated of all the approaches considered in this paper.

The intersection method is a practical approach which uses the intersection of the laminar and turbulent flow theoretical lines as the critical point (Shook & Roco, 1991). The success of this method depends on the accuracy of the turbulent model used. The Wilson and Thomas model has been used here, as this model has given good results as reported by Xu *et al* (1993). It should be emphasised that this approach is purely practical and cannot explain the flow behaviour as does the Newtonian Reynolds number approach, which works from the fundamental definition regarding inertial and viscous forces. The success of this approach is based on the empirical fact that for most non-Newtonian slurries - especially those which can be characterised using the Bingham plastic rheological model - the abrupt increase in headloss at the laminar/turbulent transition (characteristic of Newtonian flow) is absent (Shook & Roco, 1991). This method is incompatible with Newtonian behaviour, where the critical point is **not** the intersection of the laminar and turbulent theoretical lines.

One of the most recent approaches is that of Slatter (Re_3). The laminar/turbulent transition is the limit of laminar behaviour and, as such, must be compatible with the classical laminar behaviour of the slurry. However, Slatter (1995) has departed from the more traditional approaches to conceptualising the nature of the laminar flow, and rejects the plug-flow region as non-fluid behaviour. Application of this basic idea to his Reynolds number formulation (Slatter & Lazarus, 1993) yields a new pipe Reynolds number which has been shown to be more reliable than previous modelling approaches (Slatter, 1995). The flow of only the sheared fluid in the annulus is considered. The geometry of the pipe and plug are shown in Figure 1.

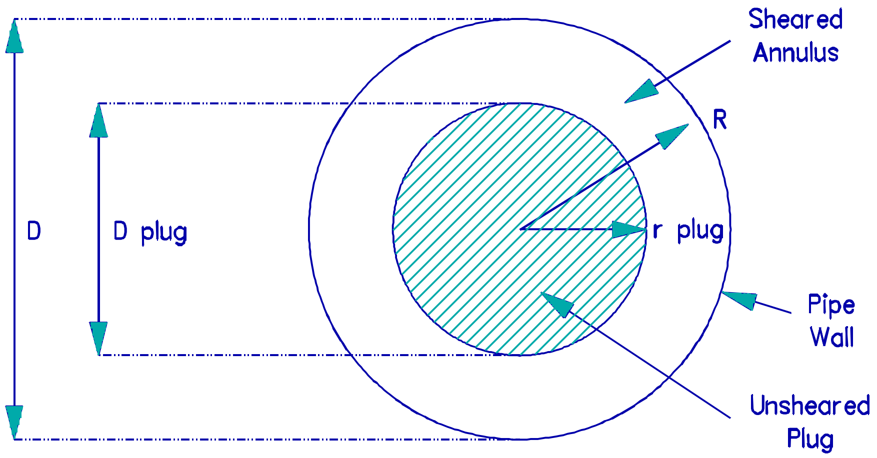

Figure 1 : Unsheared Plug Geometry

The radius of the plug is

$$r_{plug} = \frac{\tau_y}{\tau_0} R \,, \tag{11}$$

and the area of the annulus is

$$A_{ann} = \pi \left(R^2 - r_{plug}^2 \right) . \tag{12}$$

The sheared diameter, D_{shear}, is now taken as the characteristic dimension, because this represents the zone in which shearing of the material actually takes place. It is defined as

$$D_{shear} = D - D_{plug} \,, \tag{13}$$

where $D_{plug} = 2\ r_{plug}$. Since the unsheared core is treated as a solid body in the centre of the pipe, the flow which the core represents must be subtracted as it is no longer being treated as part of the fluid flow. The corrected mean velocity in the annulus V_{ann} is then obtained as follows,

$$V_{ann} = \frac{Q_{ann}}{A_{ann}} \quad \text{where} \quad Q_{ann} = Q - Q_{plug} \quad \text{and} \quad Q_{plug} = u_{plug} A_{plug} \,. \tag{14}$$

Using the same fundamental assumptions as Slatter & Lazarus (1993), the final form is

$$Re_3 = \frac{8 \rho V_{ann}^2}{\tau_y + K \left(\frac{8 V_{ann}}{D_{shear}} \right)^n} \,. \tag{15}$$

Focused analysis using an extensive experimental data base done by Slatter (1995 and 1996) has shown that this model is the most reliable over a wide range of experimental conditions.

3 ANALYSIS

Analysis of the various models reviewed above is best done from the perspective of the relationship between the critical velocity and the pipe diameter. Not only is this relationship of prime practical importance in pipeline design, but it also reveals the strengths and weaknesses of the various approaches, and their behaviour at both small and large diameter.

3.1 The Relationship Between the Critical Velocity and the Pipe Diameter

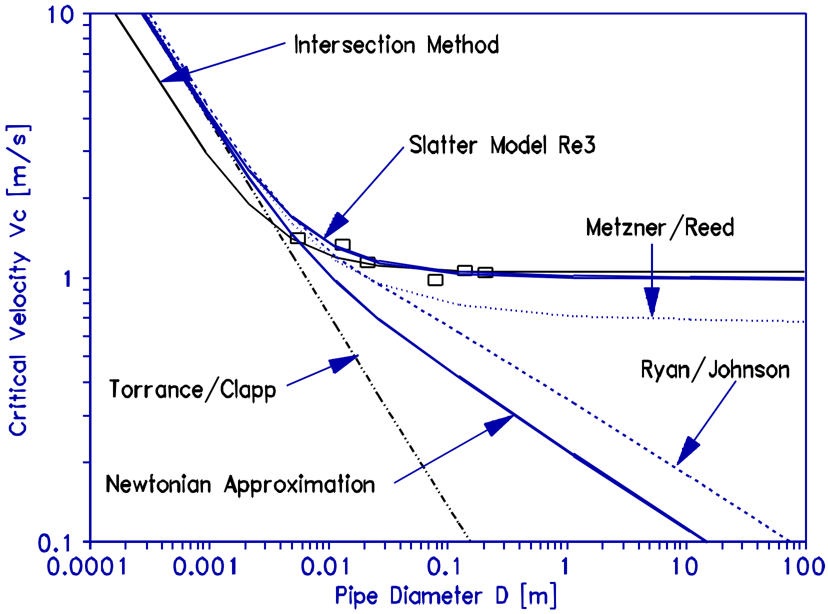

Figure 2 : Critical velocity vs pipe diameter

Figure 2 shows the theoretical models plotted against experimental data from Slatter (1995) for Kaolin slurry tests. The same slurry was tested in six different diameter pipes ranging in diameter from 5 mm to 200 mm (test set 0608; ρ = 1071 kg/m^3, τ_y = 1,88 Pa, K = 0,0102 Pa.s^n, n = 0,8428). Only the Intersection Method and the new Reynolds number Re_3 show good agreement with all the data points. Comparison of the different methods over an extensive data base (Slatter, 1995) have shown that Re_3 shows the best agreement with experimental data.

3.2 Dimensional Analysis

Although it has been pointed out (Slatter & Lazarus, 1993) that dimensional analysis should be used with caution in this area, it can be employed in order to establish the limiting conditions (high and low asymptotes) of behaviour. Starting out with

$$V_c = \Phi\ (\rho,\ D,\ \tau_y,\ K,\ n)\ , \qquad \textbf{(16)}$$

it can be shown that (Housner & Hudson, 1959)

$$\Pi_1 = \Phi^{/}\ (\Pi_2,\ \Pi_3)\ , \qquad \textbf{(17)}$$

where

$$I_1 = C_1 \frac{\tau_y}{\rho\ V_c^2} \ ;\ \Pi_2 = C_2 \frac{\rho\ V_c^{2-n}\ D^{\,n}}{K} \ ;\ \Pi_3 = C_3\ n \qquad \textbf{(18)}$$

3.3 Trends and Asymptotes at Large Diameter

Figure 2 shows that the data indicates a definite horizontal trend, as predicted by the Bingham Plastic Reynolds number. The Torrance/Clapp Reynolds number is unable to follow this at all. In fact the effect of the yield stress is shown to move all the other models upwards, on the right hand side of Figure 2, away from this oblique asymptote. Although the Newtonian approximation and the Ryan & Johnson model show some effect due to the yield stress, they do not predict it successfully and are some distance from the larger diameter data points. Figure 2 also shows that these two models (the Newtonian approximation and the Ryan & Johnson model) do not approach a horizontal asymptote at all.

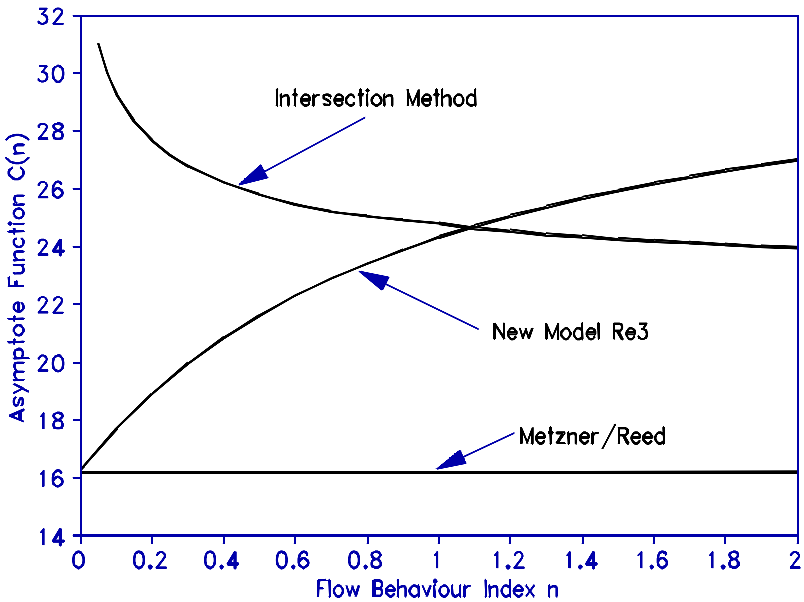

Figure 3 : Horizontal asymptote functions

In order to establish values for the horizontal asymptotes of the three remaining models, use can be made of the dimensional analysis done above. The dimensionless group Π_2 becomes insignificant as D → ∞ and the asymptote can be formulated as

$$V_{c\,\infty} = C(n)\sqrt{\frac{\tau_y}{\rho}}\ , \qquad \textbf{(19)}$$

where the function C(n) incorporates the constant values and can be plotted for the different models as shown in Figure 3.

The Metzner Reed approach asymptote value is independent of n. This can be confirmed by recasting the Reynolds number as

$$Re_{MR} = \frac{16}{f} = \frac{8\ \rho\ V^2}{\tau_0}\ . \qquad \textbf{(20)}$$

Then, noting from Equation (3) that $\tau_0 \rightarrow \tau_y$ as D → ∞ (V constant),

$$V_{c\,\infty} = \sqrt{\frac{2100}{8}\frac{\tau_y}{\rho}} = 16{,}2\sqrt{\frac{\tau_y}{\rho}}\ , \qquad \textbf{(21)}$$

which is independent of n.

As n → 0, the Slatter and Metzner Reed asymptote values converge, while that for the intersection method increases indefinitely. As n increases, the Slatter asymptote value increases while the intersection method asymptote value decreases. The Slatter and intersection method asymptote values intersect at an n value of 1,09 and in the range 0,7 < n < 1,6 the values for $V_{c\infty}$ for these two approaches are similar (less than 10% difference).

4 DISCUSSION

4.1 The effect of Rheology

At large diameter, the shear stresses required to produce a given velocity approach the yield stress. The viscous stress caused by the fluid consistency index K become insignificant and the behaviour is controlled by the yield stress and the flow behaviour index n. This produces a horizontal asymptote which is independent of both the fluid consistency index and the pipe diameter.

Most designers have a good knowledge of Newtonian fluid pipe design and are accustomed to the relationship $V_cD = 2100\mu/\rho$ which is hyperbolic (oblique with slope -1 on a log-log plot). For this case the critical velocity decreases indefinitely with increase in pipe diameter. Turbulence can thus be increased or induced by an increase in pipe diameter. The fact that the presence of a yield stress causes the critical velocity to become independent of the pipe diameter requires a paradigm shift for the pipe system designer.

The notion that turbulence can be increased or even induced by an increase in diameter does not necessarily hold true for non-Newtonian flow and must be employed with caution.

The constant value for the horizontal asymptote of the Bingham Plastic Reynolds number is 16,2 (as given in Equation (21)) and not approximately 19 as given in the literature (see Equation (8)).

Figure 2 has been plotted for a specific slurry, but the asymptotes and trends are generally applicable for $0 < n < 2$.

4.2 The Performance of the Different Approaches

As can be seen in Figure 2, at small diameter, all the approaches **-** except the intersection method **-** give reasonable agreement, agree with experimental data and will yield reliable predictions. At small diameter, the intersection method is not reliable.

At large diameter, only the Slatter, Metzner Reed and Intersection approaches are able to respond successfully to the presence of a yield stress. Focused analysis done by Slatter (1995 and 1996) has shown that this model is the most reliable over the full range of diameters. If it is accepted that this approach is the most accurate, then the Metzner Reed approach will give good results at low n values, and the intersection method will give good results in the range $0,7 < n < 1,6$. This is possibly the reason for the good performance obtained for the intersection method as employed by Xu *et al* (1993) who used the Bingham plastic model (ie $n = 1$).

The fundamental difference between the approaches revolves principally around how they accommodate the presence of the yield stress. The success of the Slatter approach can be attributed to the fact that it considers the detail of the physical behaviour of the material in the pipe as a consequence of the yield stress. All of the other Reynolds numbers considered, including the Metzner Reed approach, ignore the fact that the unsheared plug exists over a significant section of the pipe cross-section. It is therefore somewhat surprising that the Ryan and Johnson (and Hanks) criterion does not respond successfully to the presence of the yield stress, since it is scientifically the most sophisticated approach and quite clearly takes the detailed behaviour into account.

The poor performance of the Newtonian Approximation is also surprising, since the apparent viscosity is correctly evaluated at the required wall shear stress in each case. This approach is unfortunately the first choice of most designers who, understandably, feel more comfortable with the tried and trusted Newtonian approach, provided that they can obtain a value for the "viscosity". The explanation for its failure is, as explained above, because the detailed behaviour of the material across the pipe cross-section is not considered. It is insufficient to ensure that the apparent viscosity is evaluated at the correct wall shear stress, while the fact that a solid plug exists and that the velocity profile is significantly changed to accommodate it, is ignored. The detailed effect that the yield stress has on the overall response is disregarded.

A further remarkable result is the relative success of the intersection method, which has no theoretical basis at all, and owes its fortuitous success to the following explanation. Figure 2 shows that at small diameter the intersection method locus will lie below the Re_3 locus. Figure 3 shows that at large diameter and for n < 1,09 the intersection method locus will lie above the Re_3 locus. This means that for n < 1,09, these two loci must converge and intersect at intermediate diameter, as shown in Figure 2. Since it can also be seen from Figure 2 that this convergence occurs at a very acute angle, there is good agreement over a relatively large range of diameters. Most of the materials against which the performance of the intersection method has been gauged lie in this range of n, thus this approach has achieved good results.

The Torrance/Clapp Reynolds number ignores the presence of the yield stress and is not useful for the prediction of the laminar/turbulent transition. However, this approach is useful for emphasising the effect of the yield stress, and provides the asymptote for behaviour at small diameter where the effect of the yield stress is masked by the high shear stresses necessary to produce the required velocity.

Figure 2 shows that there is a limited range of intermediate pipe diameters, of approximately one decade, for which all the approaches will give reasonably accurate predictions. This would explain the surprisingly good agreement obtained by Slatter (1996). However, outside of this range, significant discrepancies will arise. Although this range is relatively small and is dependant on the slurry properties, it has been found that this often overlaps the range of experimental data reported in the literature. The reason for this is because most of the data in the literature has been collected using small and intermediate pipe diameters - there is very little data for large pipe diameters. The fact that several of the models may give reasonable agreement at small and intermediate diameters has probably lead to many confusing indications in the literature. It is believed that this is the first study of the effects of rheology and the characteristics of the various approaches over the full range of pipe diameters.

Asymptotes can easily be constructed by the designer, to delimit the extremums of behaviour. These asymptotes are also useful for showing in qualitative and quantitative terms the strengths and weaknesses of the various approaches and highlighting the characteristics of each approach.

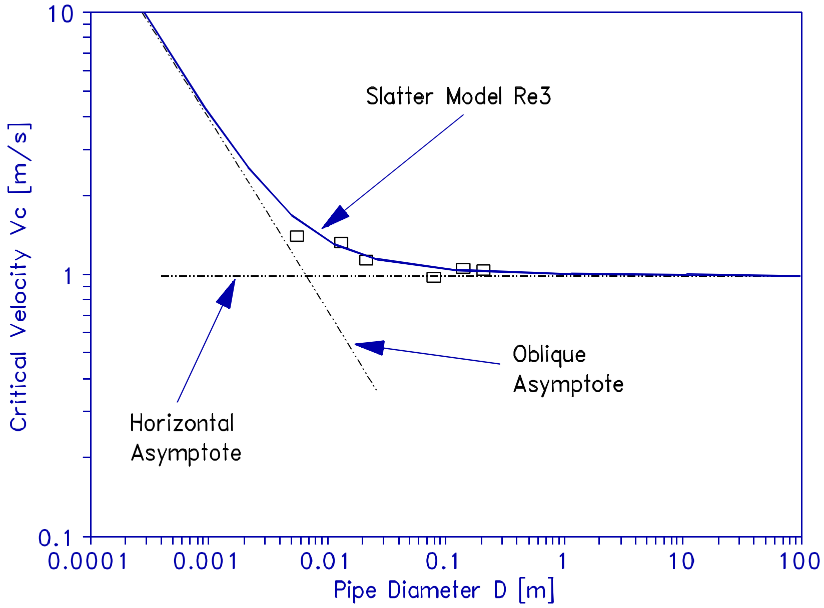

Figure 4 : The oblique and horizontal asymptotes plotted for the Slatter model Re_3

As an example of this, Figure 4 shows the oblique and horizontal asymptotes plotted for the Slatter model Re_3 and for the same slurry shown in Figure 2. The oblique asymptote is computed using Re_{nn}, and the horizontal asymptote is computed from Equation (19). The value of the function C(n) is read from Figure 3 and is 23,6.

4.3 Rheological Characterisation

The foregoing arguments emphasize the importance of the numerical values assigned to the yield stress and the flow behaviour index. At present, the values assigned are based on an engineering interpretation of the viscometric data. The rheological characterisation procedure of necessity ignores low shear rate data which may be lower than the assigned value for the yield stress. This means that the value assigned is not the true value of the yield stress, but rather a value which gives the "best fit" of all laminar flow data - deliberately excluding those points which would best indicate the true value of the yield stress. However, the final results obtained are better than those obtained by any previous method. The implication is therefore that the true value of the yield stress - if it indeed exists - is relatively unimportant and the assumptions made in the present procedure are satisfactory. This implication is of fundamental importance since some researchers have expended considerable effort in developing apparatus and ascertaining experimentally the precise value of the true yield stress.

5 CONCLUSIONS

The relationship between the critical velocity and the pipe diameter provides an ideal perspective for the investigation of the effect of rheology on the laminar/turbulent transition, as well as for the development of asymptotes for the behaviour at small and large diameter.

At small diameter, the yield stress becomes insignificant and the behaviour is controlled by K and n, producing an oblique asymptote on the V_c-D diagram which is independent of the yield stress.

At large diameter, the behaviour is controlled by the yield stress and the flow behaviour index, producing a horizontal asymptote which is independent of both the fluid consistency index and the pipe diameter. The constant value for a Bingham Plastic has been shown to be 16,2 and not approximately 19.

For materials exhibiting a yield stress, the expectation that turbulence can be increased or even induced by an increase in pipe diameter must be exercised with caution.

These trends have been shown to be generally applicable for $0 < n < 2$.

The performance of the various models can also be illustrated by separating behaviour at small and large diameter. At small diameter all the approaches, except the intersection method, will give reasonable results. Outside this range the results will not be reliable. The intersection method is not reliable.

At large diameter, the effect of the yield stress becomes paramount and the Newtonian approximation, the Torrance/Clapp approach and the Ryan and Johnson criterion (and Hanks criterion) do not respond to the yield stress effect and will not produce horizontal asymptotes. The Slatter, Metzner Reed and Intersection approaches do produce horizontal asymptotes and if it is accepted that the Slatter approach is the most accurate, then the Metzner Reed approach will give good results at low n values, and the intersection method will give good results in the range of n close to unity.

The success of the Slatter approach can be attributed to the fact that it specifically accommodates the yield stress and the phenomenon of plug flow, whereas the other Reynolds number approaches do not. The failure of the Ryan and Johnson (and Hanks) approach is surprising since it is scientifically the most sophisticated approach. Although the intersection method has no theoretical basis, it has produced good results over the experimental range of n values because it crosses all the other approaches on the V_c-D diagram at intermediate pipe diameters.

There is a limited range of intermediate pipe diameters, of approximately one decade, for which all the approaches will give reasonably accurate predictions.

Asymptotes can easily be constructed by the designer, to delimit the extremums of behaviour. These asymptotes are also useful for showing in qualitative and quantitative terms the strengths and weaknesses of the various approaches. The use of dimensional analysis has assisted in obtaining the asymptotes of behaviour.

This paper highlights the importance of the yield stress in particular. However, the rheological characterisation procedure used will not necessarily give the true value for this parameter, and there is still much debate ongoing regarding this topic. Although this debate may appear to be somewhat academic, it must be emphasised that it does have direct practical implications for the pipe system designer.

6 FURTHER RESEARCH

Although the Slatter approach Re_3 has been shown to perform well over a wide range of conditions, it has only been tested against mineral slurries and pipes up to 200 mm in diameter. Very little work is reported in the literature for pipes above 50 mm in diameter. Tests using other fluids and larger pipe diameters should be performed and the results compared with the analyses obtained, in order to confirm the trends reported above.

All the slurries tested were subjected to the same rheological characterisation procedure which ignores low shear rate data. The yield stress and the flow behaviour index are affected significantly by these data points and the final values obtained will significantly affect the predicted values for the laminar/turbulent transition. The actual values obtained and the interpretative engineering approach to viscometric data are therefore controversial and should be subjected to further debate and investigation.

No focused work has been yet been done on dilatant materials ($n > 1$). Experimental work should be pursued in this direction to confirm the findings of this investigation at higher n values.

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